**Partial differential equations**

**Homework**

**Task 10. Heat equation. Non-homogeneous equation**

**Methodical instructions**

Consider the heat transfer under the exterior heat source characterized by the given function *f.* This phenomenon is described by non-homogeneous heat equation

*ut = a2 uxx* + *f*(*x*,*t*), 0 < *x* < *L*, *t* > 0.

Suppose the initial temperature is zero. Then we have the initial condition

*u*(*x*,0) = 0, 0 < *x* < *L*.

The temperature is zero at the ends, i.e. we have the following boundary conditions

 *u*(0,*t*) = 0, *u*(*π*,*t*) = 0, *t* > 0.

**Table of parameters**

|  |  |  |  |
| --- | --- | --- | --- |
| variant | *L* | *a* | *f* |
| 1 | π | ½ | –2 sin *x* |
| 2 | 1 | 2 |  –sin π*x* |
| 3 | 1 | 3 | sin 2π*x* |

**Actions**

It is necessary perform the following steps:

1. Determine the solution of the problem as a sinus Fourier series.
2. Find the Fourier coefficient of the system parameters.
3. Solves ordinary differential equations with respect to the Fourier coefficients of the problem solution.
4. Check that this is, in reality, the solution of the boundary problem.
5. Show the graph (temperature distribution for the different time points).
6. Give the physical interpretation of the results.

Use the example from the lecture as the sample.